Mathematics in the Making 4

Rotation and Angles

Stuart E Bell

Longmans

Mathematics in the Making

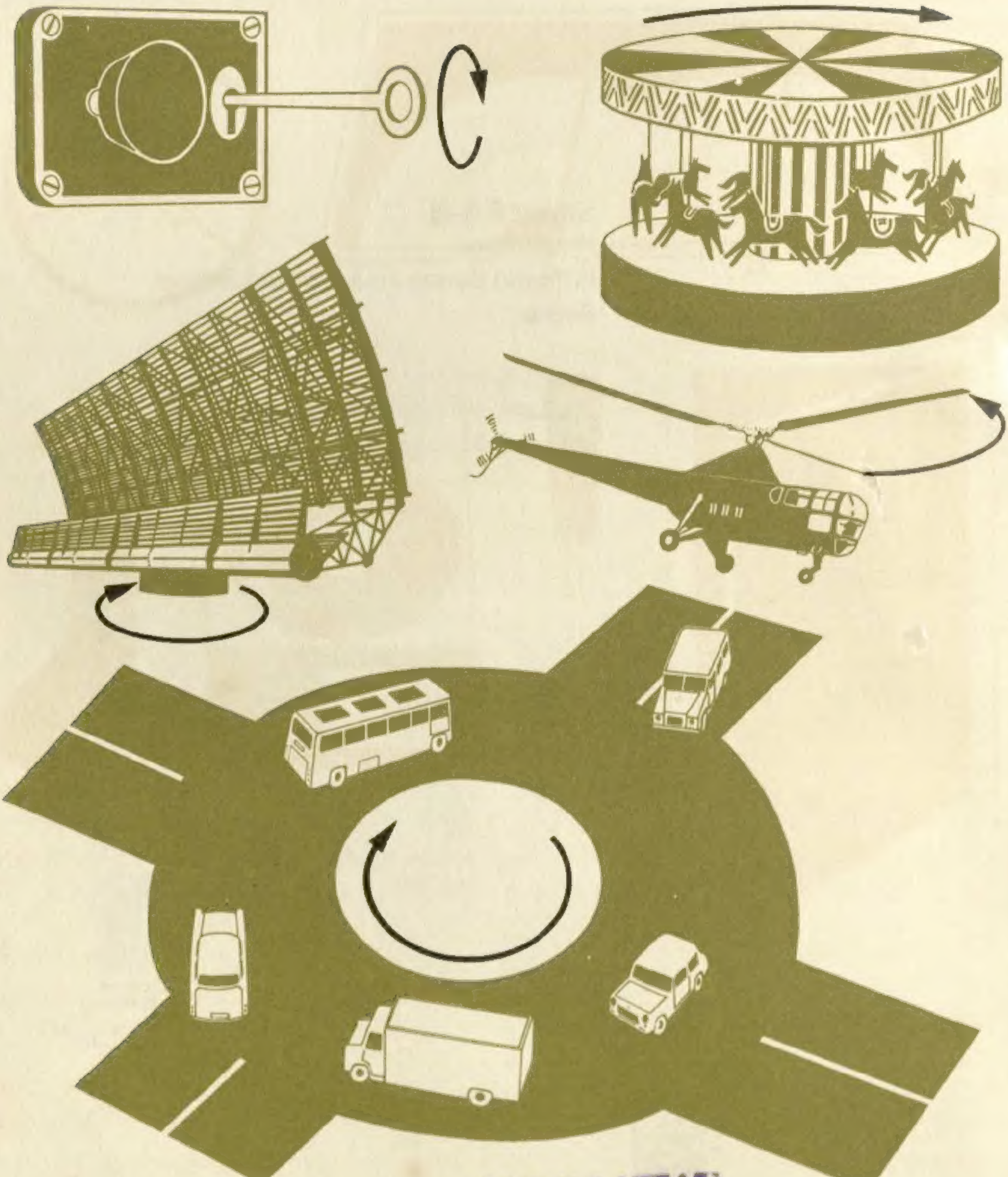
Rotation and Angles

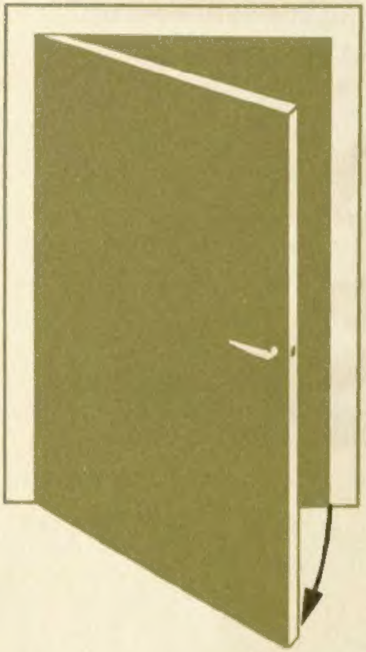
Stuart E Bell
Headmaster,
Whipton Barton Junior Mixed School,
Exeter

in collaboration with
the School Mathematics Project
at Exeter School

Longmans

Rotation and Angles





All the objects in the illustrations are turning. If you think for a while you will realise how many things in the world around us turn.
The pages of our books, the handle or knob of the door, the lid of the desk are just a few of the things in the classroom which are turned.

What else do you turn in the classroom?
Try to think of all the things you turn when you get up in the morning.
Make a list of them.

Stand up and face the front of the classroom. Turn right round until you are facing the front again.

You have now made a complete turn.
When we turn our bodies, or some object, we may move them either one complete turn, more than one complete turn, or part of a turn.

Turn to face the right.
What part of a complete turn have you made ?

Turn to the front again.
Now turn around so that you are facing the back wall.
What part of a complete turn have you made now ?
Face the front.
Make a $\frac{1}{4}$ turn right.
Make a $\frac{1}{4}$ turn left.
Make a $\frac{1}{2}$ turn right.
Make a $\frac{3}{4}$ turn left.
Which of the objects on pages 2 and 3 will make more than one complete turn ?

HOW BIG ARE THESE TURNS?

1 When you stop at the kerb and look right.

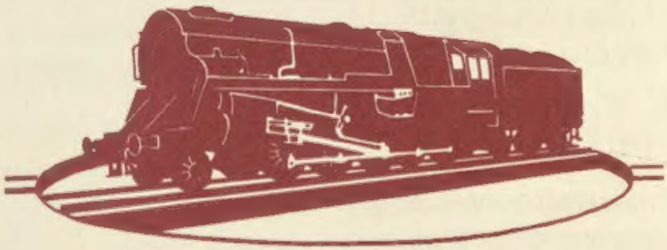
2 When you turn over a page of this book.

3 When you move the handle of the window from closed to open.

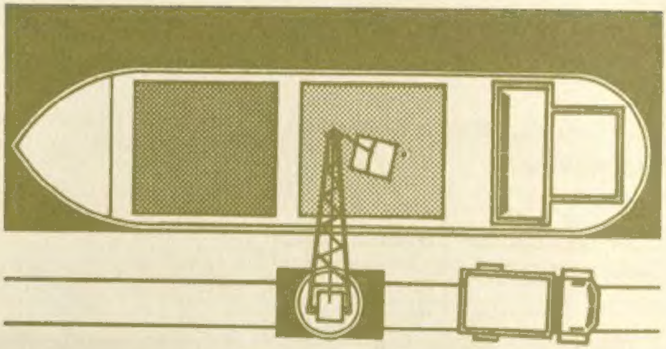
4 When you put right your watch, which has stopped at six o'clock. It is now eight o' clock. How many turns must the large hand make?

5 When you move the light switch from 'off' to 'on'.

6 When the engine is on the turntable and has to be turned in the opposite direction. How big is the turn? Can the turntable turn either way? Is it the same amount of turn either way?



How far must the crane turn to be over the lorry? Can it go either way? How big would the turn be in the other direction.



Here are some more turns for you to calculate.
Record your results in a table like the one below.

$\frac{1}{4}$ turns	$\frac{1}{2}$ turns	$\frac{3}{4}$ turns	1 complete turn	more than 1

HOW MANY TURNS OR FRACTIONS OF A TURN ARE NEEDED:

1 to wind up a watch or clock?

2 to turn the steering wheel of a car from one end of its lock to the other?

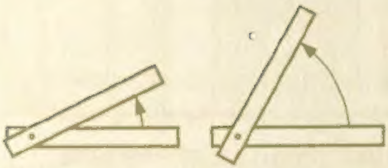
3 to turn a tap on fully?

4 to turn the volume control of your television set fully on?

5 to unlock your door with a key?

6 to unscrew the cap on your tube of toothpaste?

Angle



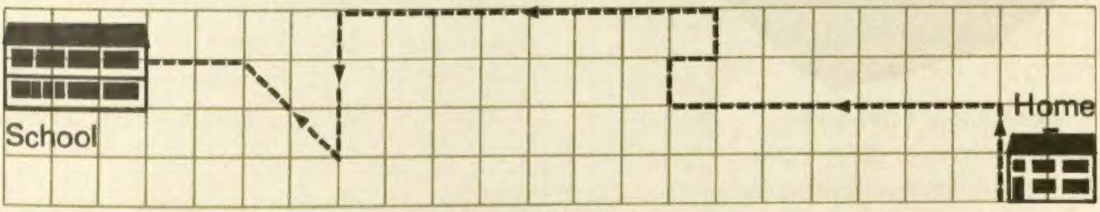
We call the amount of turn an ANGLE.
If you join two strips of cardboard together with a paper fastener and open them out like a fan you will make a series of angles getting larger all the time. You can also use two rulers in the same way.

Move your strips or rulers through the angle of a $\frac{1}{4}$ turn a $\frac{1}{2}$ turn, a $\frac{3}{4}$ turn and a complete turn.

WHAT ANGLE DOES THE LARGE HAND OF THE CLOCK TURN IN—

(a) 1 hour?
(b) 30 minutes?
(c) 15 minutes?
(d) 1 hour 30 minutes?
(e) $2\frac{3}{4}$ hours?
(f) 1 hour 45 minutes?

Here is a drawing, on squared paper, of the way a boy goes to school.



You can see all the turns the boy made and the angles he turned through.

Make a drawing of the way you come to school. Show all the turns and try to guess the angles through which you turn.

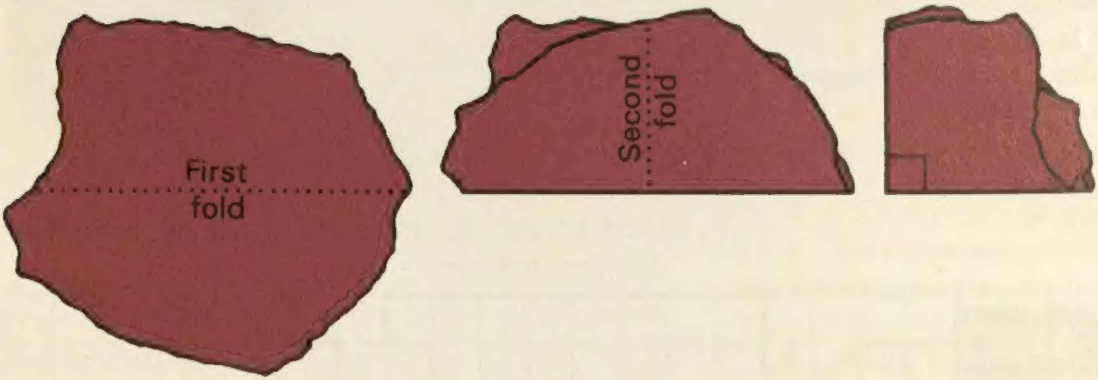
The Right Angle

How many right angles are there in a complete turn?

We call a quarter turn a RIGHT ANGLE.

We can imagine these two lines as two rulers touching. The angle between them is the angle through which one must turn to touch the other. This is shown by an arrow. The angle between the two lines is a right angle.

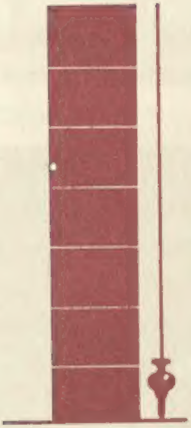
Right angles are very common and most important.
You can make a right angle by folding a piece of paper twice like this.
The piece of paper can be any shape.



Make a list of lines and surfaces in your classroom which are at right angles to each other. Use your paper right angle to help you.

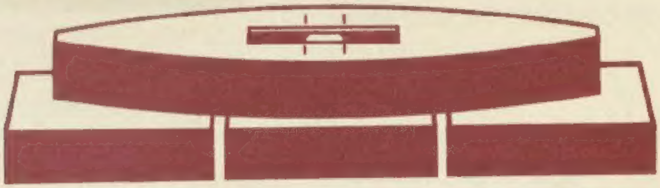
No doubt you found a great number of right angles, both in the room itself and in the furniture.

Did you find that the walls are at right angles to the floor and ceiling?



When a school or a house is being built, the workmen make sure that the walls are at right angles to the floor by using a plumb line. This is a line with a weight attached which is hung from the top of the wall. The line always hangs perfectly upright or vertical.

The workman keeps the top of the wall level or horizontal by using a spirit level. He knows when the bricks are horizontal if the bubble is in the centre of the glass window, like this.



The angle between a vertical and a horizontal line is always a right angle.

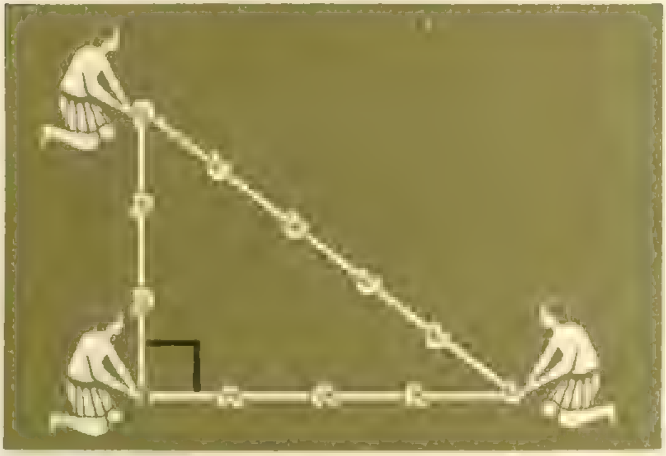
Name some lines in your classroom which are horizontal and some which are vertical. Where is a right angle formed between two horizontal lines in the room?

Long ago, builders in ancient Egypt constructed right angles with a piece of rope. They divided the rope into twelve equal parts by tying knots.

One man held the two ends of the rope together. A second man held the fourth knot from one end. A third man held the third knot from the other end.

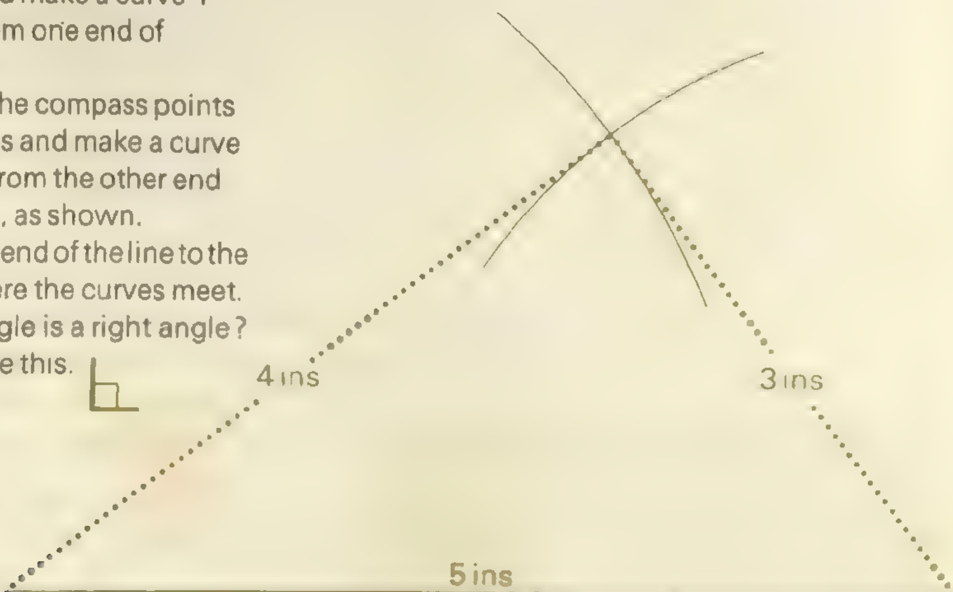
When they all pulled the rope tight they had made the shape of a triangle which contained a right angle, like this.

If the distance between each pair of knots is 1 foot, how long is each side of the triangle?
Divide a piece of string into twelve equal parts in this way and see if you obtain a right angle.
Use it to check the angles made between walls, the walls and the floor, the sides of the door, the window frames, and so on.



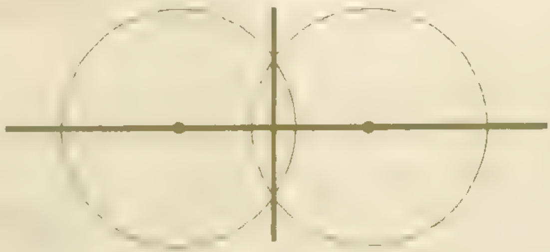
Another way of making a right angled triangle is by using a pair of compasses.

Draw a line 5 inches long.
Set the compass points to 4 inches and make a curve 4 inches from one end of the line.
Now set the compass points to 3 inches and make a curve 3 inches from the other end of the line, as shown.
Join each end of the line to the point where the curves meet.
Which angle is a right angle?
Mark it like this.



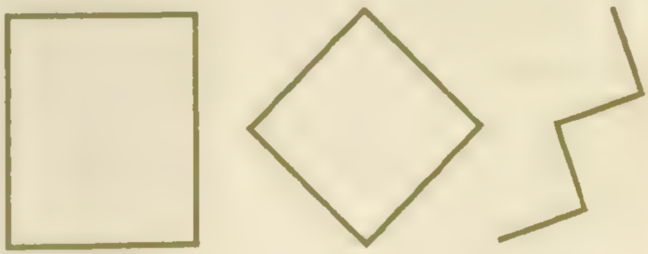
Can you discover whether you can make a right angled triangle with sides of different lengths—for instance, 4 in., 4 in., and 4 in.; or 5 in, 3 in. and 6 in.?

Here is another interesting and very accurate method of making a right angle.



Can you discover how it is done? Study the diagram carefully.
Start with the line across the page.

How many right angles can you see in each of these shapes?



A B C

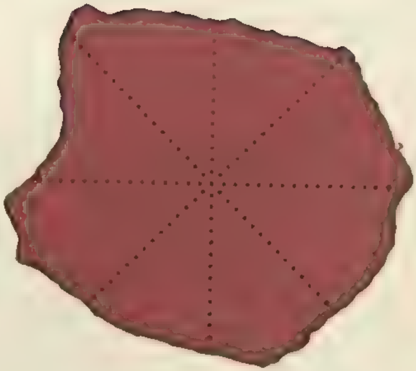
Measuring in Right Angles

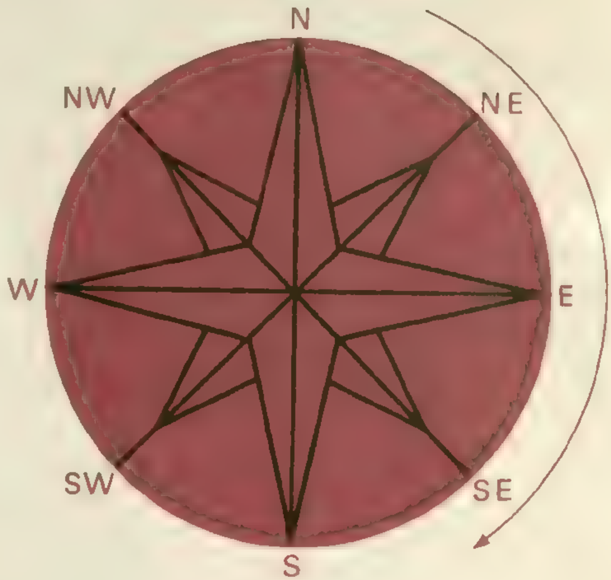
We can now measure fractions of a complete turn in right angles or parts of right angles.

Take your paper right angle and fold it again like this:

Be sure to fold the corner containing the right angle.

Open out your piece of paper and you will have the points of the compass.
Draw the compass points on the floor.
Stand facing North.
Turn through $\frac{1}{2}$ a right angle.
Turn through two right angles
Turn through three right angles.
Turn through $3\frac{1}{2}$ right angles.
Notice that two right angles make a half turn.
Mark the folds on your paper as they are on the compass.



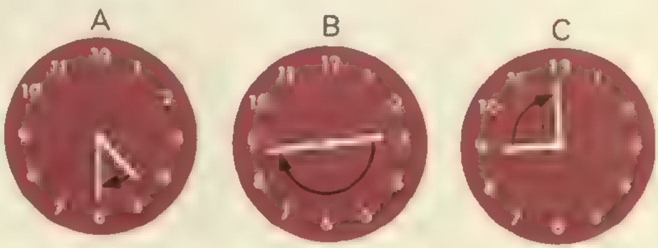


Now answer these questions

1 What is the angle between each line and the one next to it?

2 Measuring in right angles and halves of right angles, how big is the angle from:
(a) North to North East?
(b) South East to South West?
(c) South West to North East?
(d) West to North East?
(e) North West to South West?
Measure the angles in the direction of the arrow.

3 Calculate the angles between the hands of the clock in right angles and half right angles.



Acute and Obtuse Angles

Angles which are smaller than right angles are known as ACUTE angles. An acute pain is a very sharp pain, and you can see that the point made by these lines which contain an acute angle is sharp.

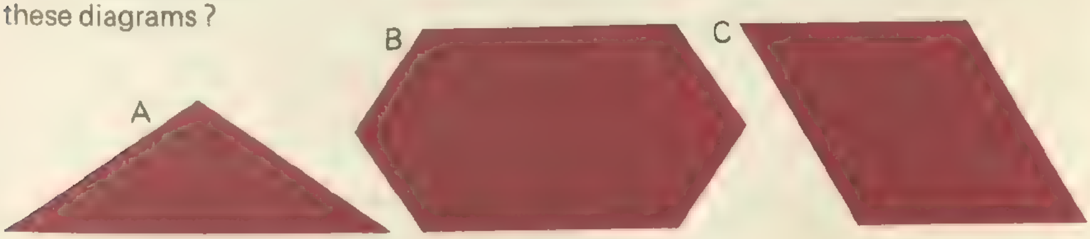
Which of these angles are acute ?

Draw two acute angles.

Angles between one and two right angles are called OBTUSE angles. Here is an obtuse angle.

Draw two obtuse angles.

2 How many obtuse angles can you see in each of these diagrams ?



Find examples of acute and obtuse angles in the classroom and school and make a list like this.

Acute Angles	Obtuse Angles

Triangles

FIND OUT THESE THINGS ON A GEO-BOARD OR PIN-BOARD.

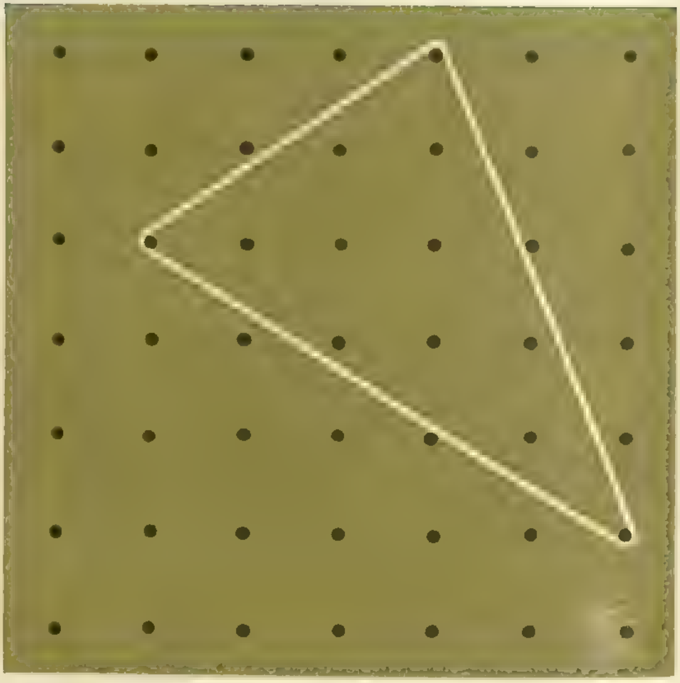
1 Can you make a triangle with three acute angles?

2 Can you make a triangle containing a right angle?

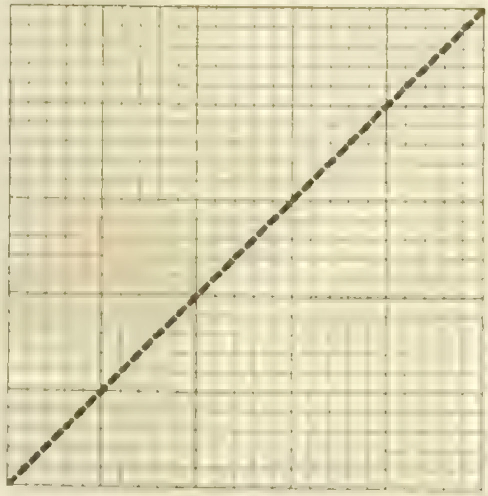
3 Can a triangle have two right angles?

4 How many obtuse angles can a triangle have?

5 Can a triangle have two equal angles?



Cut a square from a piece of graph paper.
Fold the square from corner to corner like this.



Are the triangles you have formed the same size?
What part of the square is each triangle?
Add together the angles of the square. What is the total of the four angles?
Can you now discover the total of the angles of each triangle?

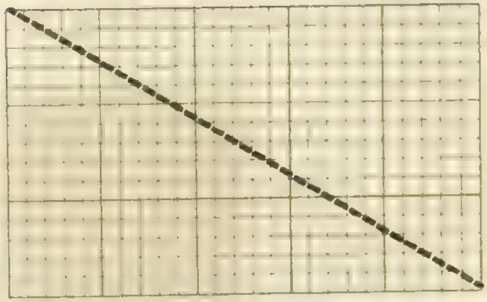
Cut a rectangle from a piece of graph paper and fold it in the same way.
Are the triangles the same size? Is each one halt the size of the rectangle?
What is the total of the angles of the rectangle?
What is the total of the angles of each triangle?

Draw a triangle and rest your pencil on the one edge so that it is pointing to the left, like this:
Turn your pencil through each angle of the triangle.
First turn it through the angle at point C.

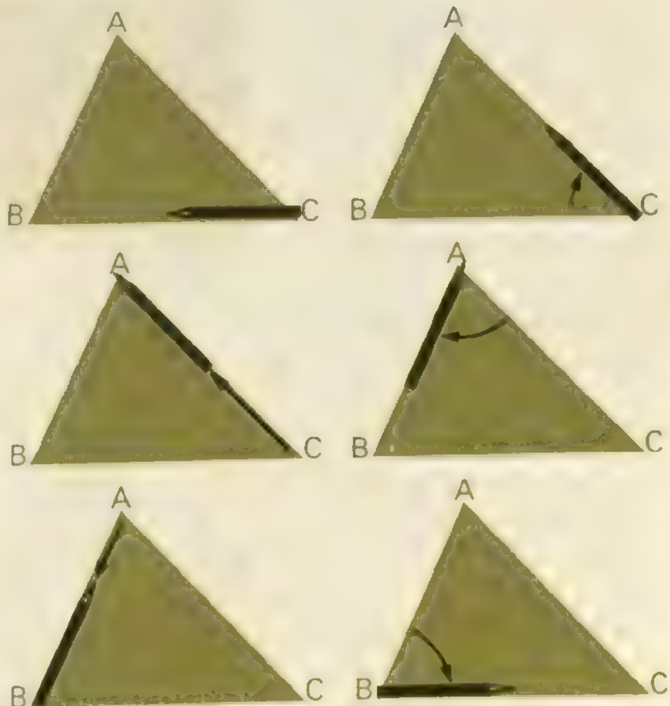
Next move the pencil on to point A and turn it through the angle there.

Lastly move the pencil to point B and turn it through the angle there.

Which way is it pointing now? What do the three angles add up to?



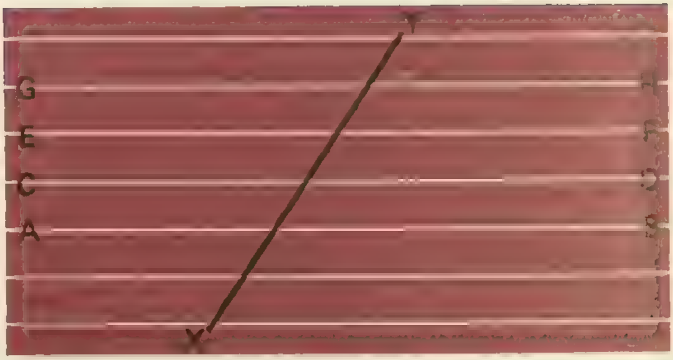
Let us find out if what we have discovered is true of every triangle.



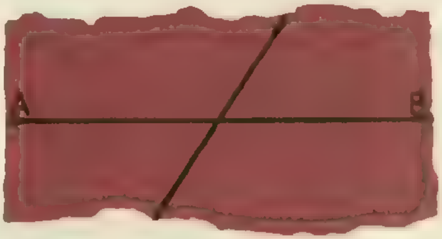
You have turned your pencil through all three angles of the triangle.

Parallel Lines

Draw a line in your exercise book at an angle to the printed lines.
Mark the lines as shown.
The lines in your text book are called PARALLEL lines because they are the same distance apart.
Can you think of other examples?

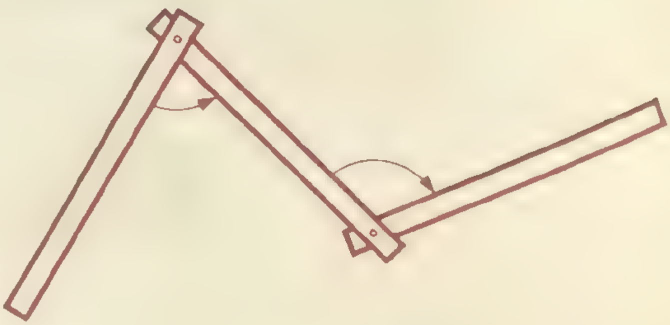


Trace and mark one crossing point on to tracing paper, like this.



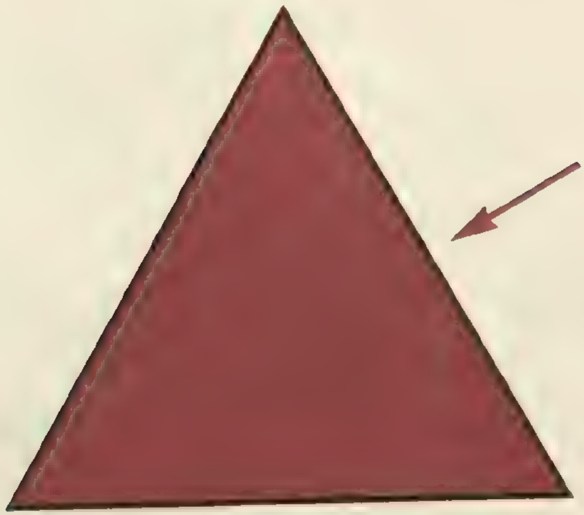
Slide the tracing paper along line XY until AB falls on CD.
What do you notice about the angles at the crossing point?
Repeat on EF, GH, and so on.
Is the result always the same?

Join three strips of cardboard together with paper fasteners, like this.



Arrange the strips so that the two angles which are marked are the same.
What do you now notice about the two outer strips?
What shape do the strips make?

A Special Triangle



Here is a special kind of triangle.

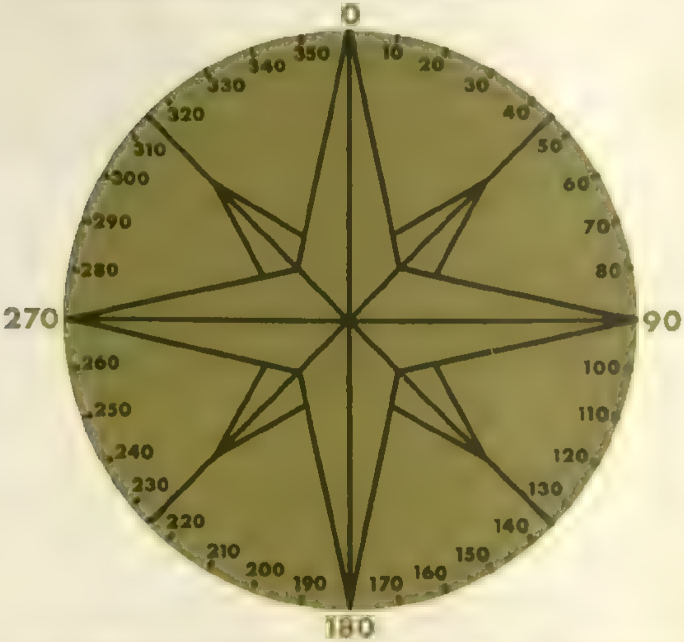
What do you notice about its sides?
What do you notice about its angles?
Are the angles acute, obtuse or right angles?
Does it look the same if we turn the page and look in the direction of the arrow?
Where have you seen this shape used?

We call this triangle an EQUILATERAL TRIANGLE because its sides and angles are equal.

Draw a line 3 inches long.
Try to discover how to make an equilateral triangle with sides 3 inches long, using a pair of compasses.

Measuring Angles in a Different Way

Sometimes we need to describe an angle exactly It would not be sufficient to tell the engineer building a television mast that his angles should be a little more than half a right angle or a little less than two right angles.
Angles can be measured very accurately in DEGREES.
Here we can see the compass marked off in degrees. There are 360 degrees in a complete turn.

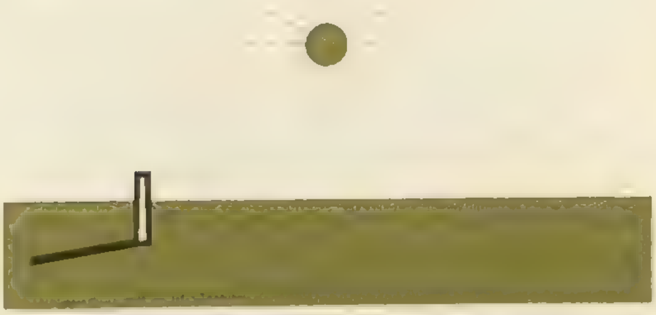


The ancient Egyptians believed that the sun moved round the earth in a circular track, taking one year of 360 days to complete the journey. So the idea that there are 360 degrees in a complete turn was accepted and used, and can be traced back several centuries before the time of Jesus Christ.

In what way were the Egyptians wrong ?

Each year was measured from the longest day of one summer to the longest day of the next by observing the shadow made by a stick at twelve o'clock noon.

Can you discover how this was done ?



1 How many degrees are there in a right angle ?

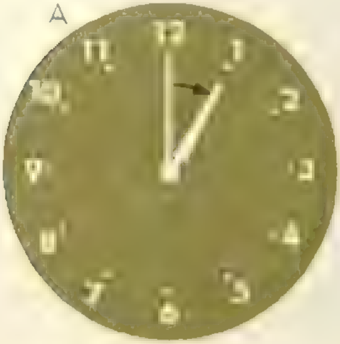
2 How many degrees are there in two right angles ?

3 How many degrees are there in half a right angle ?

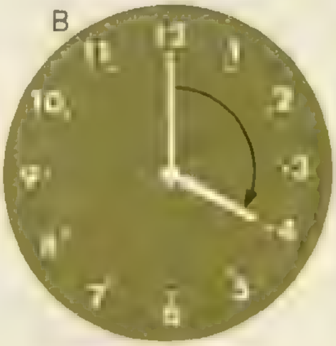
4 How many degrees are there in the three angles of a triangle ?

5 How many degrees are there between each of the hands on the clock face ?

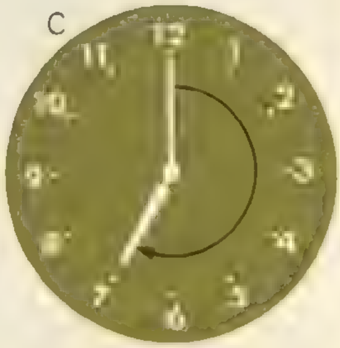
A



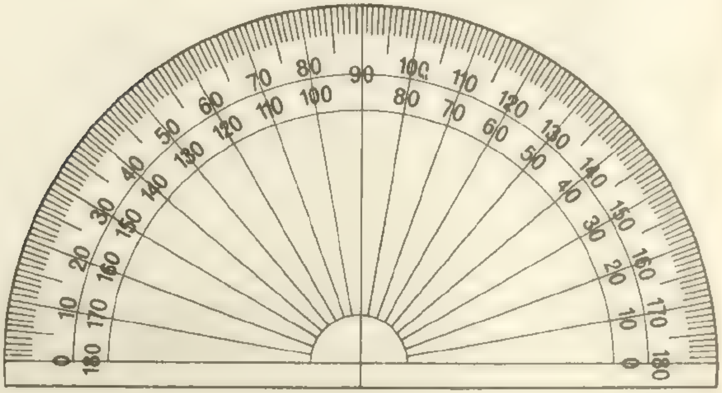
B



C

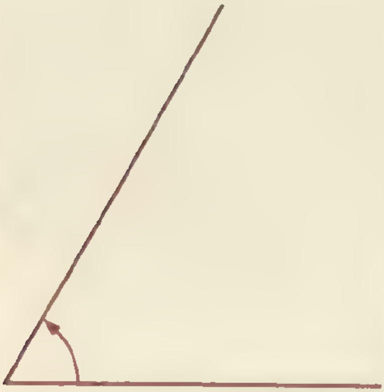


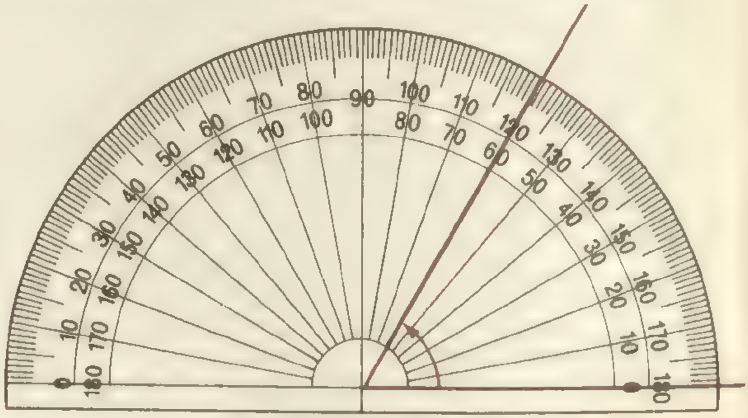
We use a protractor to measure degrees.



Notice that if we read by turning from left to right we use the numbers round the outer edge, and if we read by turning from right to left we use the inner numbers.

Here we see a protractor used to measure an angle.





We can see that the angle is 60 degrees.

How do we know it is not 120 degrees?
Using a protractor, find out how to draw an angle of 60 degrees.

A short way of writing degree or degrees is °, so 60° means 60 degrees.

Look at these angles.

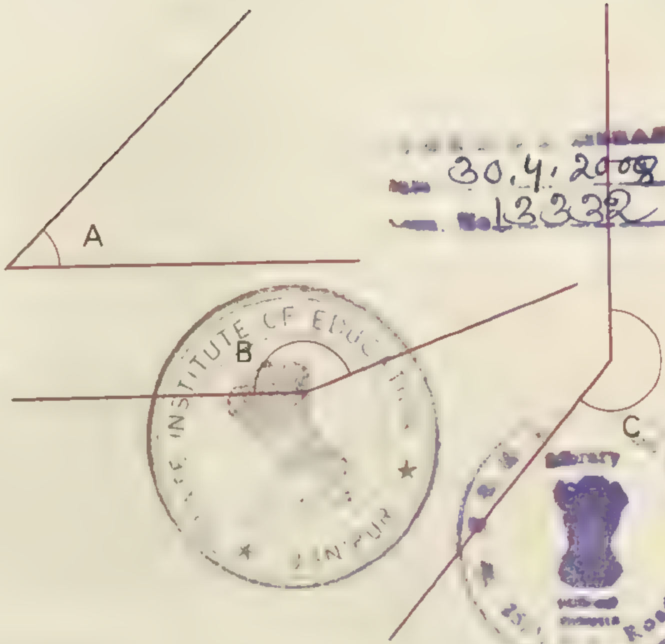
1 Estimate whether each angle is more or less than 90°.

2 Which angle is nearly 180°?

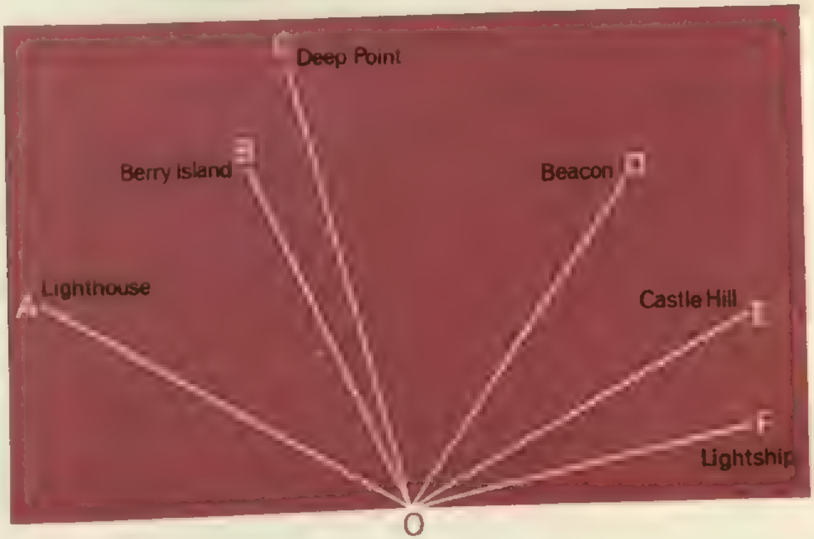
3 Measure the angles with a protractor.

4 Use a protractor to draw angles of :
(a) 45°
(b) 15°
(c) 100°
(d) 150°
(e) 180°

5 Say whether each of these angles is acute or obtuse.



Here is a picture of a table like those sometimes found on the tops of hills which point to places of interest.



What is the angle between the lines OA and OB, OB and OC, OC and OD, OD and OE, OE and OF?

The Angles of a Quadrilateral

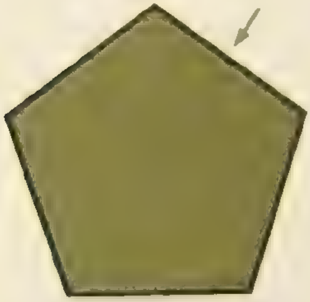
We have discovered that the total of the angles of any triangle is two right angles or 180° and the sum of the angles of a square or a rectangle is 4 right angles or 360°.
Four-sided figures like the square and the rectangle are known as QUADRILATERALS.
Here are some more quadrilaterals.

Make these with cardboard strips and paper fasteners, or on the pin board, or draw them.
What is the total of the angles of each quadrilateral?
A Clue: Join two opposite corners in each figure.

Draw figures with 5, 6, 7 and 8 sides.
Can you find the total of the angles of each figure in the same way?
Fill in a table of what you find, like this:

Number of sides	Number of triangles you made	Total of angles
3	1	180
4	2	?
5		
6		
7		
8		

The Regular Pentagon



How many sides has this shape ?

We call a five-sided figure a PENTAGON.

Where have you seen this shape used ?
What do you notice about its sides ? Are the angles equal ?
Does the shape look the same if we look from the direction of the arrow ?

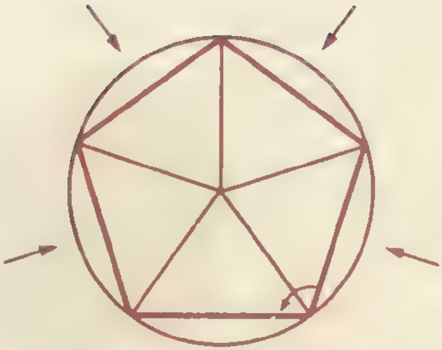
A pentagon with equal sides and angles is called a REGULAR PENTAGON.
Let us discover one way of making a regular pentagon.

Draw a circle with your compasses.

If we draw five 'spokes' equally spaced we shall find that the places where the spokes meet the circle make the five corners of the regular pentagon.

Do you remember how many degrees there are in a complete turn ?
What will be the angle between each spoke and the next one to it ?

Construct the five spokes accurately with a protractor and complete the pentagon like this:



Turn your page round and look at it from the direction of the arrows.
Does your pentagon always look the same shape?
What size will be the angle marked ?

The Regular Hexagon

A six-sided figure with equal sides and angles is called a REGULAR HEXAGON.

How many sides has this shape?
What do you notice about the sides?
Are the angles equal?

Find out if you can make a hexagon which is not regular.
Let us construct a regular hexagon in the same way that we made the regular pentagon.
First draw a circle.
How many spokes do we need this time?
What angle will there be between each spoke and the next one to it?

Draw the spokes and complete the figure.
Turn it round. Does it look the same?
Where have you seen the shape used?
You can learn more about pentagons and hexagons in *Pattern, Area and Perimeter,* another booklet in this series.

Now try to draw a regular eight-sided figure—the REGULAR OCTAGON—in a similar way.
How many spokes do you need?
What angle will there be between each spoke and the next one to it?

Can you construct a regular twelve-sided figure the same shape as a threepenny piece? How many spokes do you need? What is the angle between each spoke and the next?

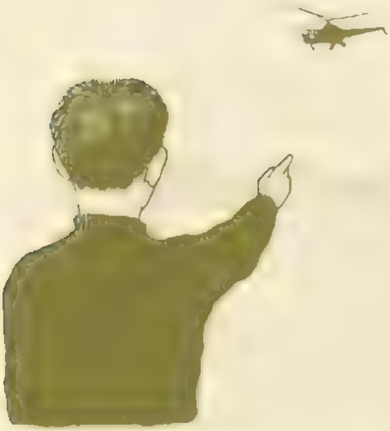
This pathway has been made from concrete slabs in the shape of hexagons.

These regular pentagons will not fit together to make a path. There are spaces between them

Why is it that the corners of regular hexagons fit together?
A clue: What is the angle of each corner of a regular hexagon?
Why is it that the corners of regular pentagons do not fit together?

Calculate whether regular 7 and 8 sided figures will fit together and fill up a space. You can check your answer by tracing these figures and trying to fit them together.

Bearings



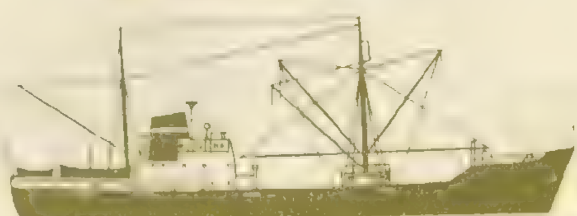
Let us suppose that you are going for a walk with your father or mother and that you suddenly see something exciting. Perhaps it is a colourful bird or animal which suddenly appears in the distance, or perhaps a helicopter.
The object is some distance away and not easy to see. How will you draw attention to it?
You will almost certainly point, but this is not always enough.

Here is a method that is more exact.

Imagine you are standing in the middle of a clock face. Look out for some prominent object, such as a tall chimney, a church tower, or a house.

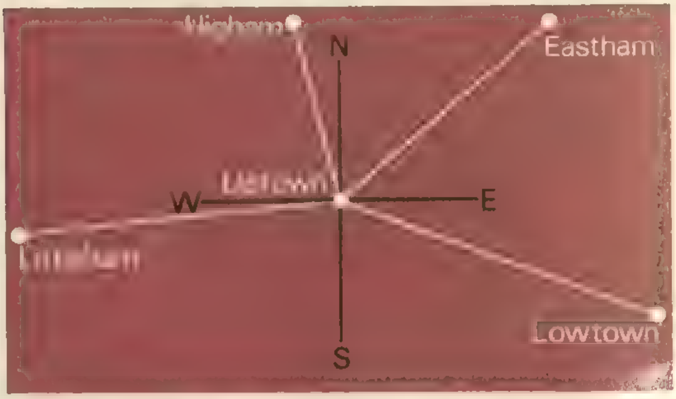
Imagine that the prominent object is at twelve o'clock on the clock face.
The helicopter is at 9 o'clock on your imaginary clock face so you say: 'From the tall chimney 9 o'clock.'
Try out this method from the classroom window or play ground.

Have you ever wondered how a ship is steered across the sea to its destination, or how an aeroplane finds its way in the dark? Neither has landmarks to help it.



The direction in which ships and aircraft travel is measured as an angle in degrees. This angle is measured round from North in a clockwise direction from 0° to 360°, like the compass on page 17. Bearings are always given in three figures so that due east is a bearing of 090°.

The bearing from UPTOWN to EASTHAM is 050°, the bearing from UPTOWN to LOWTOWN is 110°, the bearing from UPTOWN to LITTLEHAM is 264°, and the bearing from UPTOWN to HIGHAM is 346°.

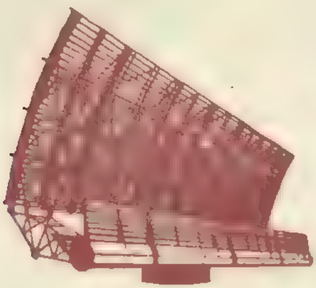


The captain of a ship or of an aircraft both read the bearing on a compass.

1 What would the direction South be as a bearing?
2 What would the direction West be as a bearing?
3 Look back at the compass points on page 18.
Give each point as a bearing.

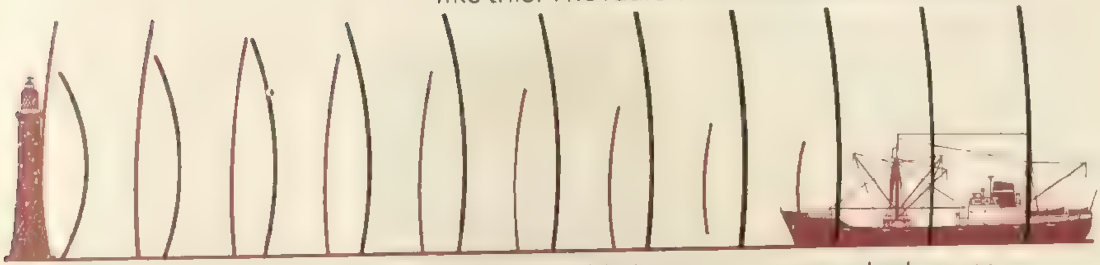
Radar

Have you seen a radar aerial on a ship or at an airport?

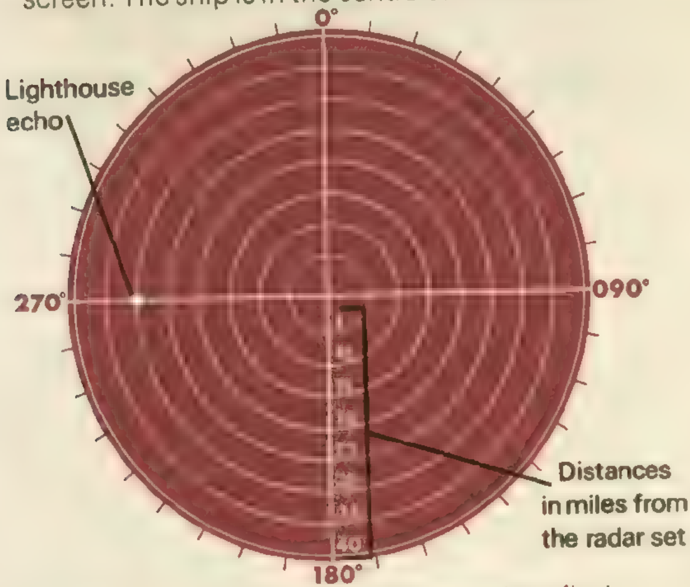


This is how it works.
The radar set sends out radio signals as the ship is moving. The signals bounce off the objects they meet (such as other ships, cliffs and buildings) as echoes in just the same way as your voice bounces back as an echo if you are standing in an enclosed place.

The radio echoes are picked up by the aerial like this. The radio echoes are shown in black.



Inside the ship is a radar screen which is very similar in appearance to a television screen. The screen is marked off not only in degrees, but in miles so that directions and distances can be read. Usually the screen is turned so that 0 is pointing to North.
The object which sends back the radio echo can be seen as a white 'blip' or line on the screen. Here we can see the lighthouse as a blip on the screen. The ship is in the centre of the screen.



1 How far away is the lighthouse?

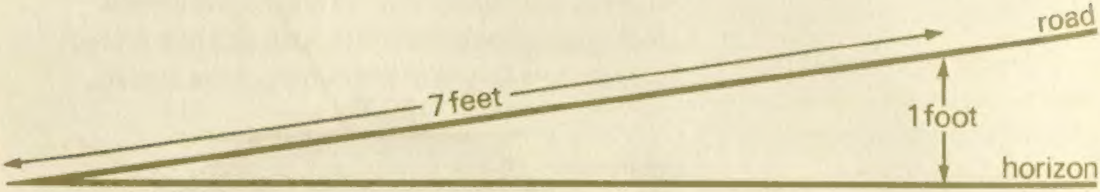
2 What is its bearing?

Radar is most useful in helping vessels to find their way in darkness or in fog, when the light from the lighthouse would not be visible.

Gradients

Have you ever seen a road sign like this?

A gradient is the amount of slope.
1 in 7 means that for every 7 feet measured along the road there is a rise of 1 foot, like this.



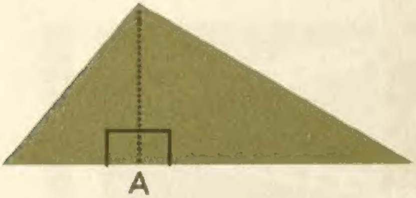
1 What does 1 in 5 mean?

2 Which is the steeper hill, 1 in 5 or 1 in 7?

3 Which is the bigger angle measured from the horizontal?
Look out for gradient signs by the roadside and on railway lines.

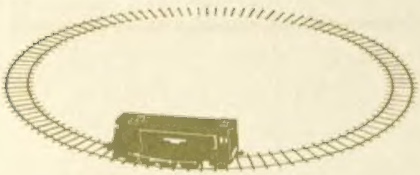
4 Which have the steeper gradients—roads or railway lines?

1 Draw any triangle.
Fold the triangle to make two right angles at A.

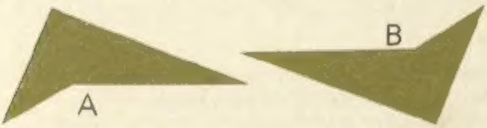


Fold the three corners of the triangle to meet at A.
What can you discover about the three angles of the triangle?

2 Through what angle does the toy train turn in going round the track 3 times?



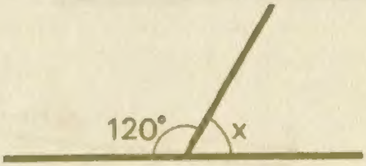
3 Look at these shapes.

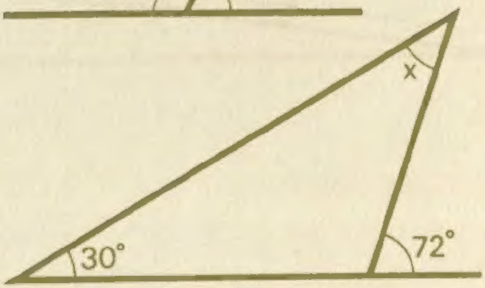


Estimate the angle you would need to turn shape A for it to be in the same position as shape B. Does it matter which way you turn the shape?

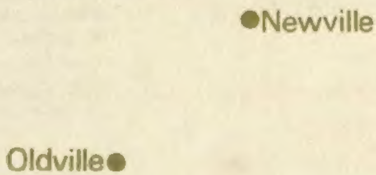
4 Without using a protractor, draw each of these angles:
(a) 45°
(b) 90°
(c) 180°
(d) 220°
Now check with a protractor. How far out were you?

5 Find the angles marked x without using a protractor.

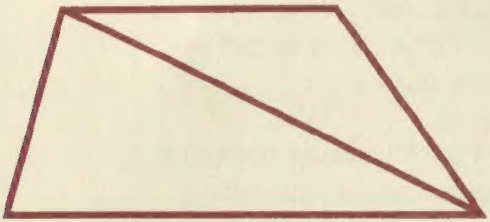




6 The bearing of Newville from Oldville is 045°. What is the bearing of Oldville from Newville?



7 Draw these shapes and mark all the angles which are equal, like this: 



8 John walks from his house to the shop and back again in the way shown by the dotted line.
Through what angle has he turned?

9 Through what angle does the earth turn in twelve hours?

Longmans, Green & Co Ltd
48 Grosvenor Street
London W1

Associated companies, branches and representatives throughout the world

First published 1967

The author has prepared these books with the help and advice of some of the members of the School Mathematics Project who have been introducing new ideas and new spirit into mathematics in the early years of the secondary school course. They feel that some of these ideas and approaches are relevant to children in the primary schools; such children would be thereby more readily equipped for the pattern of the secondary school course which is now emerging. In particular, the author gratefully acknowledges the close collaboration of Mr D J Holding of Exeter School, who is one of the principal S M P authors.

Printed in Gt. Britain by Taylowe Ltd., Maidenhead

Mathematics in the Making

Longmans